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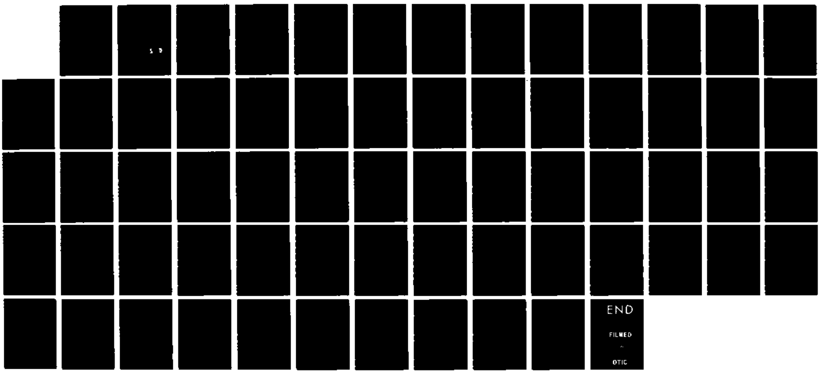

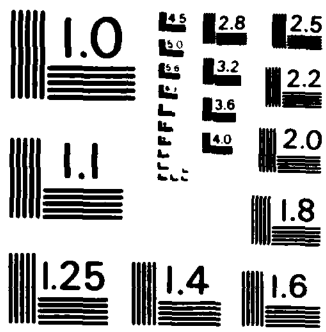

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A PIECEWISE-LINEAR APPROACH TO DC ANALYSIS OF LARGE-SCALE INTEGRATED CIRCUITS

LINGEN MAO

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A Gauss-Seidel piecewise-linear (pwl) approach to dc analysis of nonlinear circuits and systems is described. The convergence properties of the proposed method are analyzed in detail. The approach has been implemented in a computer program and successfully used to find the dc solutions of bipolar and MOS digital integrated circuits.

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A Piecewise-Linear Approach to DC Analysis of Large-Scale Integrated Circuits

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Abstract: Katzenelson's algorithm and its variants are powerful tools for solving nonlinear networks which are modeled by piecewise-linear ~~(PWL)~~ characteristics. But, when nonlinear network sizes become very large such as in VLSI chip cases, excessive cpu time and storage are required during the solution process using Katzenelson's algorithm. Decomposition techniques are necessary in the analysis of VLSI circuits. Nonlinear Gauss-Seidel iterative methods are often adopted in solving large decomposed system of equations. However, Nonlinear Gauss-Seidel iterative process will converge under certain conditions. The combination of Katzenelson and Gauss-Seidel methods proposed here takes advantages of both Katzenelson and the Gauss-Seidel methods. It decomposes the whole network into small subcircuits by Gauss-Seidel method and solves these small subcircuits by Katzenelson's algorithm separately (or even these subcircuits can be solved by Katzenelson's algorithm at same time with parallal processors, if Jacobi Method is used as decomposition technique). The convergence properties of the method is studied in detail, and examples are given here to illustrate the approach in the dc analysis of bipolar and MOS transistors circuits.

Keywords; [illegible] integrated circuits; [illegible]; computer programs; Schematic diagrams. [illegible]

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1 Introduction

The spectacular growth in scale of integrated circuit in the VLSI era needs more efficient methods for circuit simulation. In the area of computer-aided nonlinear network analysis, the techniques of piecewise-linear (PWL) approximation and analysis are efficient in computation, and have received considerable interest during the last decade. A well-known technique namely, Katzenelson's algorithm [1] which was originally applied to nonlinear resistive network with two-terminal monotonic elements, has been extended to solve more general cases [2],[3].

In piecewise-linear analysis, the nonlinear element characteristics are approximated by continuous piecewice-linear functions and the nonlinear network model can be expressed as

$$F(X) = J^{(m)}X + W^{(m)} = Y \qquad m=1,\ldots,r \tag{1}$$

where $F(X)$ is a continuous PWL mapping from R^n into itself, X is a point in R^n and represents a set of chosen variables in a given network and Y is an arbitrary point in R^n and represents the inputs, the space R^n is divided into a finite number (r) of polyhedral regions by a finite number of hyperplanes, $J^{(m)}$ is a constant $n \times n$ Jacobian matrix and $W^{(m)}$ is a constant vector defined in a given region i, and r denotes the total number of regions.

The PWL equation (1) can be solved by one of several methods: Newton's method, Katzenelson's algorithm, Jacobi iterative method, Gauss-Seidel and its extension SOR iterative methods. Newton's method suffers nonconvergence problems unless the initial guess is

close enough to the solution. Katzenelson's algorithm, which is a modified algorithm of Newton's method, is a powerful tool for solving continuous PWL equations (1) and has good convergence properties and has been refined and extended to more general cases. It has been shown that as long as all the Jacobian matrix determinants, det $J^{(m)}, m=1,\ldots,r$ in (1), have the same sign, there exists at least one solution to the equation (1) and Katzenelson's algorithm always converges to a solution [3]. Even the sign restriction was later removed in the generalized Katzenelson method [4].

The basic Katzenelson's algorithm [5] is as follows:

1. Choose an initial guess $X^{(k)}$, $k=0$
2. Find the corresponding region k and construct $J^{(k)}$ and $W^{(k)}$
3. Find $J^{(k)}X^{k} + W^{(k)} = Y^{(k)}$
4. Solve $J^{(k)}\Delta X^{(k)} = Y - Y^{(k)} = \Delta Y^{(k)}$
5. Put $X^{(k+1)} = X^{(k)} + \Delta X^{(k)}$
6. If $X^{(k+1)}$ is in region k, a soluiotn is found; otherwise, choose $X^{(k+1)} = X^{(k)} + \lambda\Delta X^{(k)}$ $(0 \leq \lambda \leq 1)$ to be on the boundary of region k and cross into adjacent region.
7. put $Y^{(k+1)} = (1 - \lambda)Y^{(k)}$ and update $J^{(k)}$ to $J^{(k+1)}$ using a dyad relationship and return to step 4.

However, when the size of a nonlinear network becomes very large, for instance, when the number of nodes $n \geq 100$, the Jacobian matrix $J^{(m)}$ in (1) becomes larger than 100 x 100 and the number of regions r may be also very large, which results in excessive cpu time and storage during the Katzenelson's procedure.

Some alternative methods for solving nonlinear equations are Gauss-Seidel, Jacobi and SOR iterations. These methods partition the n dimensional equation into n one-dimensional equations. Each equation is then solved for one variable by assuming the other (n-1) variables known. If the Gauss-Seidel method is applied to solve the equations, equation (1) can be expressed as follows

$$\begin{aligned} f_1(x_1,\ldots,\ x_i,\ldots,\ x_n) &= y_1 \\ \vdots \qquad \vdots \qquad & \vdots \\ f_i(x_1,\ldots,\ x_i,\ldots,\ x_n) &= y_i \\ \vdots \qquad \vdots \qquad & \vdots \\ f_n(x_1,\ldots,\ x_i,\ldots,\ x_n) &= y_n \end{aligned} \tag{2}$$

where f_i, x_i, and y_i are the ith component of function F(X) ,vector X and vector Y respectively. The Gauss-Seidel iterative formula [6] becomes

$$f_i(x_1^{(k+1)},\ldots,x_{i-1}^{(k+1)},\ x_i,\ldots,\ x_{i+1}^{(k)},\ x_n^{(k)})=y_i \tag{3}$$

$$x_i^{(k+1)} = x_i \tag{4}$$

When solving (3) and (4), $X^{(k)}=(x_1^{(k)},\ldots,\ x_i^{(k)},\ldots,x_n^{(k)})^T$ are known from the kth iterate and $x_1^{(k+1)},\ldots,x_{i-1}^{(k+1)}$ are also known from solving the first (i-1) components during the (k+1)th iterate. Thus (3) becomes a one-dimensional equation which is easy to solve for x_i. The process is repeated until the iterations converge, provided $\rho(J^{(m)}) < 1$, where $\rho(J^{(m)})$ is spectral radius of $J^{(m)}$ defined as the maximum modulus of the eigenvalues of matrix $J^{(m)}$ [6].

The Gauss-Seidel as well as Jacobi and SOR iterations are guaranteed to converge when any one of the following three conditions is satisfied [8]: (These conditins are sufficient but not necessary)

1. $J^{(m)}$ is symmetric and positive definite,
2. $J^{(m)}$ is a class M-matrix
3. $J^{(m)}$ is a diagonally dominant

In nonlinear transistor network analysis, the second and third conditions are more applicable since the Jacobian matrix is generally non-symmetric. A class M-matrix and diagonally dominant are defined as follows [6]:

Definition 1.1

A matrix $A \in L(R^n)$ is an M-matrix if $A=(a_{ij})$ is invertible, $A^{-1}=(b_{ij})$, and $b_{ij} \geq 0$ for all i, $j=1,\ldots,n$ and $a_{ij} \leq 0$ for all $i,j=1,\ldots,n$, $i \neq j$.

Definition 1.2

A matrix $A \in L(C^n)$ is diagonally dominant if

$$|a_{ii}| \geq \sum_{j \neq i} |a_{ij}| \qquad j=1,\ldots,n$$

and strictly diagonally dominant if strict inequality holds.

The matrices generated by the nodal approach from networks which consist of only two-terminal uncoupled resistors and independent sources are class M-matrices as well as diagonally dominant. When the circuit contains also linear two-terminal capacitors and if the capacitors are replaced by companion models using an implicit integration formula its nodal matrix is also an M-matrix. However, in many transistor networks, definition 1.1 is not satisfied; for

instance, in the TTL NAND gate shown in Fig.1. In order to form the Jacobian matrix for the TTL NAND gate in Fig.1, we use the Ebers-Moll model shown in Fig.2 to replace each npn bipolar transistor in Fig.1. Each diode in Fig.2 is modeled by piecewise-linear characteristics shown in Fig.3 (a) so that the PWL model for npn bipolar transistor which is shown in Fig.3 (b) is obtained. The sign matrix in Fig.4 is the sign of elements in the Jacobian matrix of the TTL NAND gate in Fig.1, where $\pm$ means the element in this position could be negative or positive depending on the piecewise region in which the nonlinear resistors are located; such as, the node voltages v_1=4.8, v_4=4.9, v_5=5.0, others=0.0, the signs of the elements in row 4 and column 2, and row 5 column 4 are positive, so the Jacobian matrix is not a class M-matrix. The Jacobian matrices are not always diagonally dominant either, since controlled sources or feedback loops may exist in some circuits. The point Gauss-Seidel iteration may not be converge when applied to solve this kind of circuits.

Consider a nonlinear network, such as the one in Fig.5, which consists of TTL NAND gates. The network matrix is neither an M-matrix nor diagonally dominant, and the nonlinear point Gauss-Seidel iteration does not converge to a solution. On the other hand, Katzenelson's algorithm when applied to solve this network converges to the solution. But, as mentioned earlier, applying Katzenelson's algorithm on the entire circuit would require a relatively excessive computational time and storage. In some cases even when nonlinear point Gauss-Seidel method converges to a solution, excessive computa-

tional time is required before the solution is reached. In this case, a block Gauss-Seidel iterative method could converge to the solution in less time.

In this paper a PWL block Gauss-Seidel approach is proposed. The approach combines Katzenelson's algorithm and Gauss-Seidel iteration for solving large scale nonlinear networks which are modeled by PWL functions. For example, the entire network in Fig.5 is first decomposed into small subnetworks (or, subcircuits) as shown in Fig.1. The subcircuits are solved one at a time in a given determined sequence. The method will be refered to as Katzenelson-Gauss-Seidel method, which will be described in detail in section 3. In section 2, we first introduce some basic properties of nonlinear point Gauss-Seidel iteration.

2 Convergence conditions for nonlinear Point Gauss-Seidel iteration

In this section, we introduce some definitions and theorems and give the convergence conditions for nonlinear point Gauss-Seidel iteration followed by the convergence conditions for Katzenelson-Gauss-Seidel method. When considering convergence conditions, certain measures for comparing vectors and matrices are needed. Comparing vectors in R^n element by element has been found to be one of the most useful measures. This can be done by means of the natural (or componentwise) partial ordering on R^n defined by

For $X,Y \in R^n$, $X \leq Y$, if and only if $x_i \leq y_i$ $i=1,\ldots,n$

where X, Y are vectors on R^n and x_i, y_i are the ith components of X, Y respectively. Similarly, for matrices

For A, B $\in L(R^n, R^m)$, $A \leq B$, if and only if $a_{ij} \leq b_{ij}$

$i=1,\ldots,m,\ j=1,\ldots n$

where a_{ij} and b_{ij} are the elements in row i and column j in matrices A and B. From above, $A \geq 0$ means that $a_{ij} \geq 0$, and A is called a nonnegative matrix. We use $F: D\ R^n \to R^m$, $f_i(X)$, $i=1,\ldots,m$, $X \in R^n$, as a notation for a mapping F with domain D in R^n and range F(D) in R^m and with components $f_1,\ldots,f_m$. The following are some definitions [6] which are necessary for understanding the convergence conditons of the Gauss-Seidel iteration.

Definition 2.1

the mapping $F: D \subset R^n \to R^m$ is isotone (or antitone) (on R^n) if $X \leq Y$ for any X, Y $\in$ D, implies that $F(X) \leq F(Y)$ (or $F(X) \geq F(Y)$); if the inequality holds, F is strictly isotone (or antitone).

Definition 2.2

the mapping $F: D \subset R^n \to R^m$ is inverse isotone (on R^n) if

$F(X) \leq F(Y)$ for any X, Y $\in$ D, implies that $X \leq Y$

Definition 2.3

the mapping $F: D \subset R^n \to R^m$ is diagonally isotone (on R^n) if for any $X \in R^n$ the functions

$$\phi_{i,i}: \{t \in R^1 \mid X+te^i \in D\} \to R^1,\quad \phi_{i,i}(t)=F_i(X+te^i),\ i=1,\ldots,n$$

are isotone, where e^i is a unit basic vector with the ith componont one and all others zero. Analogously, we can get

Definition 2.4

the mapping $F: D \subset R^n \to R^m$ is off-diagonally antitone (on R^n) if for any $X \in R^n$ the functions

$$\phi_{i,j}: \{t \in R^1 \mid X+te^j \in D\} \to R^1,\ \phi_{i,j}(t)=F_i(X+te^j)\quad i=j, i,j=1,\dots,n$$

are antitone. where e^j is a unit basic vector with the jth componont one and all others zero. The following gives the definition of the M-function, which is an extension of the M-matrix to the nonlinear case.

Definition 2.5

the mapping $F: D \subset R^n \to R^n$ is an M-function if F is inverse isotone and off-diagonally antitone.

The M-matrix defined in section 1 has the properties that the diagonal elements are positive and the off-diagonal elements are negative and the inverse of M-matrix is nonnegative. The M-function has similar properties. The following theorem describes some properties of M-functions.

Theorem 2.1

Let $F: D \subset R^n \to R^n$ be an M-function (and hence injection) ,then F and $F^{-1}: F(D) \subset R^n \to R^n$ are strictly diagonally isotone. If $F: R^n \to R^n$ is continuous and surjective, then F and $F^{-1}: R^n \to R^n$ are surjectively diagonally isotone.

The proof can be found in [6].

With these definitions and theorem, we can state the convergence properties of nonlinear Gauss-Seidel iteration.

<u>Theorem</u> 2.2

Let $F: D \subset R^n \rightarrow R^n$ be continuous off-diagonally antitone and strictly diagonally isotone. Suppose that for some $B \in R^n$ there exist points X^o, $Y^o \in R^n$ such that

$$X^o \leq Y^o, \quad F(X^o) \leq B \leq F(Y^o)$$

Then, the Gauss-Seidel iterates $\{Y^k\}$ $\{X^k\}$ given by (3) and (4) and starting from Y^o and X^o, are uniquely defined and satisfy

$$X^o \leq X^k \leq X^{k+1} \leq Y^{k+1} \leq Y^k \leq y^o, \quad F(X)^k \leq B \leq F(Y)^k \qquad k=0,1,\ldots \tag{5}$$

as well as

$$\lim_{k\rightarrow\infty} X^k = X^* \leq Y^* = \lim_{k\rightarrow\infty} Y^k, \; F(X^*) = F(Y^*) = B.$$

This proof can be found in [6], and is given in the Appendix for easy reference.

<u>Theorem</u> 2.3

Let $F: R^n \rightarrow R^n$ be a continuous M-function from R^n onto itself. Then for any $B \in R^n$, any starting point X^o, the Gauss-Seidel iterates (3) converge to the unique solution X^* of $F(X)=B$.

The proof is also given in the Appendix. What has been discussed so far is suitable for general continuous nonlinear functions, which include continuous PWL functions. The following theorem deals with continuous PWL mappings:

<u>Theorem</u> 2.4

Let F: $R^n \rightarrow R^n$ be a continuous piecewice-linear affine mapping (1), for which all matrices {J} are M-matrices, then F is an M-function.

This theorem is proved in [10].

By theorems 2.3 and 2.4, as long as the matrices {J} in piecewise mapping (1) are M-matrices, nonlinear Gauss-Seidel iteration can be used in PWL cases and converges to the unique solution.

Another condition which guarantees the convergence of point Gauss-Seidel iteration is diagonal dominance. If a matrix A is diagonally dominant, and A can be split as:

$$A = D - L - U$$

where D, L and U are the diagonal part, lower triangular part and upper triangular part of A respectively. then the sequence generated by the point Gauss-seidel iterative method in solving the linear matrix equation

$$AX = P$$

can be written as:

$$X^{k+1} = (D - L)^{-1}UX^k + (D - L)^{-1}P \tag{6}$$

provided (D - L) is nonsingular.

The point Gauss-Seidel iterative sequence $\{X^k\}$ in (6) will converge to the unique solution by the Diagonal Dominance Theorem in [7] since the matrix A is diagonally dominant.

For piecewise linear cases, if the matrices $J^{(m)}$ in (1) are diagonally dominant and can be split into:

$$J^{(m)} = D^{(m)} - L^{(m)} - U^{(m)}$$

Let

$$M^{(m)} = (D^{(m)} - L^{(m)})^{-1}U^{(m)},$$

and

$$P^{(m)}=(D^{(m)} - L^{(m)})^{-1}(Y - W^{(m)})$$

provided $(D^{(m)} - L^{(m)})$ is nonsingular. If the Gauss-Seidel iterative method is used, equation (1) can be rewritten as follows:

$$X^k = M^{(m)}X^{k-1} + P^{(m)} \tag{7}$$

where $M^{(m)}$ and $P^{(m)}$ have different values depending on the variable values in X. If M_i and P_i are used to express $M^{(m)}$ and $P^{(m)}$ which correspond to the (i-1)th iterative vector X^{i-1}, equation (7) can be rewritten as:

$$X^k = M_kX^{k-1} + P_k \tag{8}$$

The following theorem is given for diagonal dominance cases.

Theorem 2.5

If all of the Jacobian matrices in (1) are diagonally dominant, the Gauss-Seidel iterates of equation (8) converge to the unique solution for any initial point X^0

Proof

Assuming X^0 is an initial guess, by (7), we obtain:

$$X^1 = M_1X^0 + P_1$$

$$X^2 = M_2X^1 + P_2$$

$$= M_2(M_1X^0+P_1) + P_2$$

$$= M_2M_1X^0 + M_2P_1 + P_2$$

$$\cdots,$$

$$X^k = M_k X^{k-1} + P_k$$
$$= M_k M_{k-1} \cdots M_2 M_1 X^o + M_k M_{k-1} \cdots M_2 P_1 + M_k M_{k-1} \cdots M_3 P_2$$
$$+ M_k M_{k-1} \cdots M_4 P_3 + \cdots + M_k P_{k-1} + P_k \qquad (9)$$

There exists a norm which satisfies $||M|| = \rho(M)$, where $\rho(M)$ is the spectral radius-the maximum modulus of the eigenvalues of the matrix M [6]. In term of the norm, (9) will be:

$$||X^k|| = ||M_k M_{k-1} \cdots M_2 M_1 X^o + M_k M_{k-1} \cdots M_2 P_1 + M_k M_{k-1} \cdots M_3 P_2$$
$$+ M_k M_{k-1} \cdots M_4 P_3 + \cdots + M_k P_{k-1} + P_k||$$

By the properties of the norm

$$||X^k|| \leq ||M_k M_{k-1} \cdots M_2 M_1 X^o|| + ||M_k M_{k-1} \cdots M_2 P_1||$$
$$+ ||M_k M_{k-1} \cdots M_3 P_2|| + ||M_k M_{k-1} \cdots M_4 P_3||$$
$$+ \cdots + ||M_k P_{k-1}|| + ||P_k||$$

and

$$||X^k|| \leq ||M_k||\ ||M_{k-1}|| \cdots ||M_2||\ ||M_1||\ ||X^o||$$
$$+ ||M_k||\ ||M_{k-1}|| \cdots ||M_2||\ ||P_1||$$
$$+ ||M_k||\ ||M_{k-1}|| \cdots ||M_3||\ ||P_2||$$
$$+ ||M_k||\ ||M_{k-1}|| \cdots ||M_4||\ ||P_3||$$
$$+ \cdots + ||M_k||\ ||P_{k-1}|| + ||P_k||$$

suppose $||M_m||$ is the maximum norm among $\{ ||M_k|| \}$, and $||P_m|| < \infty$ is the maximum norm among $\{ ||P_k|| \}$, the following expression can be obtained:

$$||X^k|| \leq ||M_m||^k ||X_o|| + ||P_m|| \left(||M_m||^k + ||M_m||^{k-1} + \cdots, + ||I|| \right)$$

Since the all of the matrices $J^{(m)}$ are diagonally domin.nt, the spectral radius $\rho(M^{(m)}) < 1$ by Diagonal Dominance Theorem [7].

Because $\rho(M_m) \leq 1$, so that $||M_m|| \leq 1$, and it follows that

$$\lim_{k \to \infty} M_m^k = 0$$

$$\lim_{k \to \infty} ||X^k|| \leq ||P_m|| \, (\, ||I|| - ||M_m|| \,)^{-1}$$

The Gauss-Seidel iterative sequence $\{X^k\}$ is bounded and converges to the unique solution.

3 Convergence Condition for Block-Katzenelson-Gauss-Seidel method

The motivation for adopting the block Gauss-Seidel iterative method is to obtain convergence when the point Gauss-Seidel iterative method does not converge, and to improve the convergence rate (however, in some cases the block iterative method is slower than point iterative method [9]). As long as at least one of the diagonal submatrices A_{ii} contains a nonzero entry from above the main diagonal of A, the block Gauss-Seidel iterative method must have a larger asymptotic rate of convergence than that of point Gauss-Seidel iterative method [9].

The Katzenelson-Gauss-Seidel method to be discussed in this section is a nonlinear block Gauss-Seidel iterative method where the blocks and subvectors are considered as basic units and are solved by Katzenelson's algorihtm.

In the PWL analysis of very large-scale systems and circuits, the Jacobian matrices {J} of (1) can be partitioned into

$$J = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ A_{m1} & A_{m2} & \cdots & A_{mm} \end{bmatrix}$$

where the diagonal blocks A_{ii}, $i=1,\ldots,m$, which are assumed to be square and nonsingular matrices, correspond to subcircuits, while the nonzero elements in the off-diagonal blocks represent the connection relationship between the diagonal blocks (subcircuits).

The vectors W, X and Y of (1) are partitioned according to the partitioning of Jacobian matrices; the matrix equation (1) can then be written as

$$F = \begin{bmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \cdot & & & \cdot \\ \cdot & & & \cdot \\ A_{m1} & A_{m2} & \cdots & A_{mm} \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ X_m \end{bmatrix} + \begin{bmatrix} W_1 \\ W_2 \\ \cdot \\ \cdot \\ W_m \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \\ \cdot \\ \cdot \\ B_m \end{bmatrix} \tag{10}$$

where X_i, W_i, $B_i \in R^{n_i}$, $i=1,\ldots,m$ are the ith subvectors of X, W, Y $\in R^n$ and $\sum_{i=1}^{m} n_i = n$

The mapping F_i which is the ith subset of F can be further expressed in the form

$$F_i(X) = A_{ii}X_i + W_i + \sum_{j=1}^{m} A_{ij}X_j = B_i, \; j \neq i \tag{11}$$

Before we discuss the convergence conditions of Katzenelson-Gauss-Seidel method for solving equation (10) some new definitions for block Gauss-Seidel iteration method are needed. These definitions are created by analogy to the point Gauss-Seidel case and are stated below.

Definition 3.1

The mapping F: D $R^n \to R^n$ is diagonal block isotone (on R^n) if for any $X \in R^n$ the functions

$$\phi_{i,i}: \{t \in R^1 | X+tE^i \in D\} \to R^{n_i}, \quad \phi_{i,i}(t)=F_i(X+tE^i), \; i=1,\ldots,m$$

are isotone. where E^i is a unit basic vector with ith subvector one and all others zero.

Definition 3.2

The mapping F: D $R^n \to R^n$ is off-diagonal block antitone (on R^n) if for any $X \in R^n$ the functions

$$\phi_{i,j}: \{t \in R^1 | X+tE^j \in D\} \to R^{n_j}, \quad \phi_{i,j}(t)=F_i(X+tE^j) \; i \neq j, i,j=1,\ldots,m$$

are antitone. where E^j is a unit basic vector with the jth subvector one and all others zero.

Definition 3.3

The mapping F: D $R^n \to R^n$ is inverse isotone (on R^n) if $F(X) \leq F(Y)$ for any $X,Y \in R^n$ implies that $X \leq Y$. Here $X,Y \in R^n$ are divided into subvectors $X_i \in R^{n_i}$, $i=1,\ldots,m$, $\sum_{i=1}^{m} n_i = n$.

With these new definitions, we now prove the following theorem.

Theorem 3.1

Let the PWL continuous mapping $F: D\ R^n \rightarrow R^n$ in (10) be, (a) off-diagonal block antitone, (b) strictly diagonal block isotone, and (c) inverse isotone, then F is defined as block M-function and the Katzenelson-Gauss-Seidel method converges to the unique solution.

<u>Proof</u>:

The Katzenelson-Gauss-Seidel method formula is the following

$$F_i(X_1^{k+1},\ldots,X_{i-1}^{k+1},X_i,X_{i+1}^{k},\ldots,X_m^{k})=B_i \tag{12}$$

$$X_i^{k+1} = X_i \tag{13}$$

where X_i, $B_i \in R^{n_i}$, $i=1,\ldots,m$ are the ith subvectors of X, $B \in R^n$, $\sum_{i=1}^{m} n_i=n$, and the X_j^k means the subvector X_j of the k-th iterate.

(i) for Katzenelson's algorithm part

Here we use Katzenelson's algorithm only to solve the subcircuits; that is, the diagonal blocks, separately. We can consider that equation (12) is the same as equation (11). Let $J_i=A_{ii}$, then (11) can be rewritten as:

$$F_i(X_i) = J_iX_i+W_i = C_i$$

$$C_i = B_i-\sum_{j=1}^{m} A_{ij}X_j, \quad j\neq i$$

When the Gauss-Seidel iterative method is applied to the matrix equation such that $X^k=(X_1^k,\ldots,X_m^k)^T$ and X_j^{k+1}, $j=1,\ldots,i-1$ are given from previous Gauss-Seidel iterate and the first i-1 step in this iterate, so that C_i in (11) is considered constant. Then, F_i is a piecewise-linear mapping corresponding to subcircuit i, and Katzenelson's algorithm is applied to solve the subcircuit.

There are already a number of papers dealing with the convergence properties of Katzenelson's algorithm. We just mention some conclusions here. Katzenelson's algorithm was originally applied to the nonlinear network consisting of resistors with continuous, piecewise-linear, strictly isotonic characteristics [1]. Later, it was proved that if the determinants of $J^{(m)}$ in (1) had the same sign in all the regions, then Katzenelson's algorithm was guaranteed to converge to a solution. In this paper we only deal with the circuits which consist of the nonlinear resistors with continuous piecewise-linear strictly isotonic characteristics so that the Katzenelson's algorithm is guaranteed to converge to a unique solution.

(ii) for Gauss-Seidel part:

Suppose the starting points are X^o, $Y^o \in R^n$, and

$$X^o = (X_1^o,\dots,X_i^o,\dots,X_m^o)^T$$
$$Y^o = (Y_1^o,\dots,Y_i^o,\dots,Y_m^o)^T$$

and V^* is the solution,

$$V^* = (V_1^*,\dots,V_i^*,\dots,V_m^*)^T$$

where X_i, Y_i and V_i^*, $i=1,\dots,m$ are subvectors corresponding to subcircuit i. Since F is block isotone, the following relations are satisfied

$$F(Y^o) \geq F(V^*) \geq F(X^o), \text{ means that, } Y^o \geq V^* \geq X^o$$

By induction, for some $K \geq 0$, $i \geq 1$,

$$X^o \leq X^k \leq V^* \leq Y^k \leq Y^o, \quad F(X^k) \leq B \leq F(Y^k) \tag{14}$$

$$X_j^k \leq X_j^{k+1} \leq V_j^* \leq Y_j^{k+1} \leq Y_j^k, \quad j=1,\dots,i-1 \tag{15}$$

For $k=0, i=1$, (14) holds, and $j=0$ in (15), so that (15) is empty. Suppose (14) and (15) hold for k and $i-1$.

By the off-diagonal block antitone property, it follows that

$$P(S) = F_i(X_1^{k+1},\ldots,S,X_{i+1}^k,\ldots) \tag{16}$$

$$Q(S) = F_i(Y_1^{k+1},\ldots,S,Y_{i+1}^k,\ldots) \tag{17}$$

then, $$Q(S) \leq P(S) \qquad S \in R^{n_i} \tag{18}$$

and

$$P(X_i^k) \leq F_i(X^k) \leq B_i \leq F_i(Y^k) \leq Q(Y_i^k)$$
$$P(V_i^*) \geq B_i = F_i(V_1^*,\ldots,V_i^*,\ldots,V_m^*) \geq Q(V_i^*) \tag{19}$$

Let $P(\bar{X}_i^k) = B_i = Q(\bar{Y}_i^k)$, and by the diagonal block isotone and (19), we can obtain

$$X_i^k \leq \bar{X}_i^k \leq V_i^*$$

and

$$V_i^* \leq \bar{Y}_i^k \leq Y_i^k$$

For Gauss-Seidel method

$$Y_i^{k+1} = \bar{Y}_i^k$$
$$X_i^{k+1} = \bar{X}_i^k$$

then,

$$X_i^k \leq X_i^{k+1} \leq V_i^* \leq Y_i^{k+1} \leq Y_i^k$$

holds for $i=1,2,\ldots,m$, and hence,

$$X^k \leq X^{k+1} \leq V^* \leq \ Y^{k+1} \leq Y^K.$$

Then, we obtain,

$$F_i(Y^{k+1}) \geq \ F_i(Y_i^{k+1},\ldots,Y_i^{k+1},Y_{i+1}^k,\ldots) = B_i.$$

Similarly,

$$F_i(X^{k+1}) \leq B_i$$

Hence, this completes the induction of (14) and (15), and the sequences {X} and {Y} obtained from Gauss-Seidel iterate have the limits

$$\lim_{k\to\infty} X^{k+1} = X^*$$

and

$$\lim_{k\to\infty} Y^{k+1} = Y^*$$

and the following formulas:

$$F(X^*) \leq B, \text{ and } F(Y^*) \geq B$$

By the continuous Mapping , it follows that $F(X^*)=F(Y^*)=B$, and $X^*=Y^*=V^*$.

<u>Remark</u> 3.1

In the above proof, diagonal block isotone means the mapping F_i is isotone in the diagonal block i with respect to subvector X_i. Actualy, it is not necessary that the mapping F_i be isotone with respect to the entire subvector X_i. For instance, assume the mapping F_i in (11) can be partitioned into two components F_{i1} and F_{i2}:

$$F_{i1}=A_{rr}X_r + A_{rs}X_s \quad \sum_{j=1}^{m} A_{rj}X_j + W_r = B_r \tag{20}$$

$$F_{i2}=A_{sr}X_r + A_{ss}X_s \quad \sum_{j=1}^{m} A_{sj}X_j + W_s = B_s \tag{21}$$

$j=1,\dots,m,\ j\neq i$

where X_r, X_s and B_r, B_s are the components of X_i and B_i:

$$X_i = (X_r,X_s)^T,\ B_i = (B_r,B_s)^T,$$

and A_{ii} is partitioned into:

$$A_{ii} = \begin{matrix} A_{rr} & A_{rs} \\ A_{sr} & A_{ss} \end{matrix}$$

and X_i, $B_i \in R^{n_i}$, X_r, $B_r \in R^{n_r}$, X_s, $B_s \in R^{n_s}$, $n_r+n_s=n_i$.

Assume also that X_s is related to the other subvectors through the off-diagonal blocks and while X_r is not related to the other subvectors, that means A_{rj}, $j=1,\dots,m$, $j\neq i$, are zero, and (20) becomes:

$$A_{rr}X_r + A_{rs}X_s + W_r = B_r$$

so that X_r can be expressed as:

$$X_r = -A_{rr}^{-1} A_{rs}X_s + A_{rr}^{-1}(B_r-W_r) \tag{22}$$

Substituting (22) into (21), we obtain:

$$F_i'=A_{ii}'X_s + \sum_{j=1}^{m} A_{sj}X_j + W_s = B_i', \quad j\neq i \tag{23}$$

where

$$A'_{ii}= A_{ss} - A_{sr} A_{rr}^{-1} A_{rs}$$

$$B'_i = B_s - A_{rr}^{-1} (B_r - W_r)$$

We can now use (23) instead of (11), during the Gauss-Seidel iteration. Then, it is sufficient to have F_i' isotone with respect to subvector X_s instead of X_i for convergence.

<u>Remark</u> 3.2

It is not necessary that the F be inverse isotone with respect to the entire vector X either. In order to explain this, we still use the above equation (20), (21), (22), and (23). By eliminating X_r that is not related to the other subvectors through the off-diagonal blocks, matrix A_{ii} becomes A'_{ii} and subvector X_i becomes $X'_i=X_s$ and F_i becomes F'_i. Similarly, the other subvectors X_j, $j=1,\dots,m$,

become X'_j, j=1,...,m, and the diagonal blocks A_{jj} become A'_{jj}. Then a new mapping F' is obtained which consists of F'_i, i=1,...,m. It is sufficient that mapping F' be inverse isotone with respect to subvectors X'_i i=1,...,m for convergence. Using X'_i and A'_{ii} instead of X_i and A_{ii}, namely, using the mapping F' instead of the mapping F, Theorem 3.1 still holds.

Remark 3.3

It is possible that the convergence of the Katzenelson-Gauss-Seidel method can be speeded up by using SOR iteration instead of standard Gauss-Seidel iteration, that is, using

$$X_i^{k+1} = X_i^k + \omega(X_i - X_i^k), \quad 0 < \omega < 2, \tag{24}$$

instead of equation (4).

Another factor which affects the convergence rate is the absolute values of the off-diagonal elements. The smaller the absolute values of the upper-triangular elements in the whole matrix,the faster the convergence will be. This proof can be found in [7].

So far, we have only discussed the properties of mapping F. We show now what kind of matrices will ensure that F in (10) is block M-function.

Definition 3.4

A matrix A is block M-matrix if the matrix A can be partitioned into blocks, such that (a) all of the elements in off-diagonal blocks are negative or zero, (b) each diagonal block is diagonally dominant with each diagonal element positive (c) each diagonal block can be tranformed into a class M-matrix by means of the method in Remark 3.1

and 3.2.

<u>Theorem</u> 3.2

If the matrices {J} in (1) or in (6) are block M-matrices, then the Katzenelson-Gauss-Seidel method is convergent.

<u>Proof</u>:

Clearly, from the condition (a) and (b) in definition 3.4, it follows that F is off-diagoanl block antitone and diagonal block isotone. According to condition (c), each diagonal block becomes M-matrix after some elements are eliminated. The inverse of an M-matrix is a positive matrix which ensures the diagonal block to be inverse isotone with respect to those variables which have relationships with the other diagonal blocks. Since the diagonal elements are dominant in the entire matrix and the off-diagoanl elements are negative, the entire matrix is an M-matrix after each diagonal block is transformed into an M-matrix by means of eliminating the variables that have no relationship with the other blocks. Then with the Remark 3.2, the mapping F′ is inverse isotone with respect to the new subvectors and F′ is block M- function and the Katzenelson-Gauss-Seidel method is convergent.

The other case is when the entire matrix and its diagonal blocks are not diagonally dominant, then by using the transformation formulas in Remarks 3.1 and 3.2, the new reduced matrix and its diagonal blocks become diagonally dominant and the block Gauss-Seidel method becomes convergent.

The following are examples where the block Gauss-Seidel method

converges, while the point Gauss-Seidel does not.

Example 1

Consider the matrix equation:

$$\begin{bmatrix} 3 & 7 & 0 & 0 \\ 8 & 4 & 0 & -3 \\ 0 & 0 & 5 & -3 \\ 0 & -2 & 8 & 6 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix} = \begin{bmatrix} 5 \\ 4 \\ 8 \\ 10 \end{bmatrix}$$

If the point Gauss-Seidel method is used to solve this equation, the following formula is obtained:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}^{k+1} = \begin{bmatrix} 3 & 0 & 0 & 0 \\ 8 & 4 & 0 & 0 \\ 0 & 0 & 5 & 0 \\ 0 & -2 & 8 & 6 \end{bmatrix}^{-1} \cdot \left(\begin{bmatrix} 0 & -7 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}^{k} + \begin{bmatrix} 5 \\ 4 \\ 8 \\ 10 \end{bmatrix} \right)$$

in which case $\rho((D-L)^{-1}U)=4.6$ and the iterates will not converge. If the block Gauss-Seidel method is adopted to solve this equation and the 4x4 matrix is partitioned into 2x2 blocks, the following formula is obtained:

$$\begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}^{k+1} = \begin{bmatrix} 3 & 7 & 0 & 0 \\ 8 & 4 & 0 & 0 \\ 0 & 0 & 5 & -3 \\ 0 & -2 & 8 & 6 \end{bmatrix}^{-1} \cdot \left(\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \\ X_3 \\ X_4 \end{bmatrix}^{k} + \begin{bmatrix} 5 \\ 4 \\ 8 \\ 10 \end{bmatrix} \right)$$

The spectral radius of the bolck Gauss-Seidel matrix is 0.341, which is less than 1, so that the block Gauss-Seidel method is convergent. The new reduced matrix generated from the tramsformation described above is

$$\begin{bmatrix} -14.66 & -3 \\ -2 & 10.8 \end{bmatrix}$$

which is diagonally dominant.

Example 2

Given the matrix:

$$\begin{bmatrix} 3 & -7 & 0 & 0 \\ 8 & 4 & 0 & -3 \\ 0 & 0 & 5 & -3 \\ 0 & -2 & 8 & 6 \end{bmatrix}$$

It can be transformed into:

$$\begin{bmatrix} 22.66 & -3 \\ -2 & 10.8 \end{bmatrix}$$

by using block Gauss-Seidel method. The reduced matrix is a class M-matrix and a matrix equation consists of this matrix can be solved by block Gauss-Seidel iterative method.

In general the Katzenelson-Gauss-Seidel method can be used on a wider class of problems than having the reduced matrix digonally dominant or class M matrix. For simplicity, we only discuss the cases in which matrices are partitioned into 4 blocks. For instance, A_1, A_2, A_3, A_4 are blocks of

the main matrix A:

$$A = \begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix}$$

A_1,A_4 are diagonal blocks, and A_2, A_3 are off-diagonal blocks.

Consider a linear equation consisting of the matrix A and the vectors $X=(X_1,X_2)^T$, $B=(B_1,B_2)^T$, as follows:

$$\begin{bmatrix} A_1 & A_2 \\ A_3 & A_4 \end{bmatrix} \begin{bmatrix} X_1 \\ X_2 \end{bmatrix} = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

Applying block Gauss-Seidel method, we obtain the following:

$$\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}^{k+1} = \begin{bmatrix} A_1 & 0 \\ A_3 & A_4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} 0 & -A_2 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} X_1 \\ X_2 \end{bmatrix}^{k} + \begin{bmatrix} A_1 & 0 \\ A_3 & A_4 \end{bmatrix}^{-1} \cdot \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{25}$$

or, $$X^{k+1} = MX^k + P \tag{26}$$

where M and P are respectively:

$$M = \begin{bmatrix} A_1 & 0 \\ A_3 & A_4 \end{bmatrix}^{-1} \begin{bmatrix} 0 & -A_2 \\ 0 & 0 \end{bmatrix}$$

$$P = \begin{bmatrix} A_1 & 0 \\ A_3 & A_4 \end{bmatrix}^{-1} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

<u>Theorem</u> 3.3

If the maximum modulus of the eigenvalues of matrix M in (26) is smaller than 1, the block Gauss-Seidel iterative process converges to the unique solution.

Proof:

Assuming X^o is initial guess, by (26), we obtain:

$$X^1 = MX^o + P$$

$$X^2 = MX^1 + P = M(MX^o + P) + P$$

$$= P + MP + M^2X^o,$$

$$\ldots,$$

$$X^{k+1} = MX^k + P$$

$$= P + MP + M^2P + \ldots, + M^{k-1}P + M^kX^o$$

$$= P(I + M + M^2 + \ldots, + M^{k-1}) + M^kX^o$$

By Newman Lemma [6], if $\rho(M) < 1$, then

$$\lim_{k \to \infty} \sum_{i=0}^{k} M^k = (I-M)^{-1}$$

and

$$\lim_{k \to \infty} M^k = 0$$

such that

$$\lim_{k \to \infty} X^{k+1} = P(I-M)^{-1}$$

X^{k+1} converges to the unique solution.

The matrix M is the product of

$$\begin{bmatrix} A_1 & 0 \\ A_3 & A_4 \end{bmatrix}^{-1} \begin{bmatrix} 0 & -A_2 \\ 0 & 0 \end{bmatrix}$$

and becomes

$$M = \begin{bmatrix} 0 & -A_1^{-1}A_2 \\ 0 & A_4^{-1}A_3A_1^{-1}A_2. \end{bmatrix}$$

This is equivalent to requiring that the maximum modulus of the eigenvalues of $A_4^{-1}A_3A_1^{-1}A_2$ is less than 1. This is the basic convergence condition for block Gauss-Seidel method in the linear case. In this case, the matrix A need not necessarily be of class M-matrix or diagonally dominant.

In piecewise-linear cases, the elements of A_1, A_2, A_3, and A_4 will change their values according to the values of the variables. If the block Katzenelson-Gauss-Seidel method is used, equation (26) should be rewritten as:

$$X^{k+1} = M_{k+1}X^k + P_{k+1} \tag{27}$$

where M_k and P_k are variable depending on the piecewise linear region k which the vector X^k is located in.

<u>Theorem</u> 3.4

A piecewise-linear mapping F in (1), or (10) can be split and expressed as in (27) when block Gauss-Seidel iterative method is applied; if the spectral radii $\rho(M_k)$ are less than 1, $k=1,\dots,r$, then Katzenelson-Gauss-Seidel method converges.

The proof is similar to that of Theorem 2.5 except that the matrices 'M_k's are different. The M_k is the block Gauss-Seidel matrix here, while the M_k in theorem 2.5 is the point Gauss-Seidel matrix.

4 Examples

<u>Example</u> 1

The first example is a bipolar circuit which is shown in Fig.5, whose subcircuits are TTL NAND gates as the one shown in Fig. 1. The bipolar transistors are expressed by Ebers-Moll model shown in Fig.2, and the PWL function in Fig.3(a) represents approximately the exponential function for diode characteristics in Ebers-Moll model. Then the PWL model which replaces each npn bipolar transistor in PWL analysis is obtained in Fig.3(b). The PWL characteristics is strictly monotonic and expressed by seven segments, and each segment is described by four parameters (start breakpoint, slope, intercept and endbreak point). The parameters of the diode characteristics are given in table 1.

Table 1

segment	end breakpoint	slope	intercept
1	-0.1	0.1755e-10	-0.4580d-7
2	0.1	0.4581e-6	0.0e+0
3	0.3	0.5127e-3	-0.5122e-4
4	0.5	0.1124e+1	-0.3371e+0
5	0.7	0.2462e+4	-0.1231e+4
6	0.9	0.5396e+7	-0.3777e+7
7	∞	0.1182e+11	-0.1064e+11

where the slopes and intercepts are taken respectively as conductances and independent current sources in the equivalent circuits. The parameters α_f=0.99, α_r=0.5 for the Ebers-Moll model in Fig.2 are considered constant.

The circuit in Fig.5 is partitioned into 4 subcircuits as shown in Fig.6(a), where small resistors are inserted in such a way that

each subcircuit input and output nodes will not be connected to other subcircuits directly; rather they are connected to a set of new nodes: n1 ~ n5. Since the resistors inserted are the same, the relations of new node voltages to the subcircuit input and output node voltages are

$$
\begin{aligned}
&v_{n1}=(v_7+v'_9+\tilde{v}_8)/3,\\
&v_{n2}=(v_8+v'_8)/2,\\
&v_{n3}=(v_9+\tilde{v}_9)/2,\\
&v_{n4}=(v'_7+\bar{v}_8)/2,\\
&v_{n5}=(\tilde{v}_7+\bar{v}_9)/2.
\end{aligned}
\tag{28}
$$

where $v_7 \sim v_9$ belong to first subcircuit, $v'_7 \sim v'_9$ belong to the second subcircuit, $\tilde{v}_7 \sim \tilde{v}_9$ belong to the third subcircuit, and $\bar{v}_7 \sim \bar{v}_9$ belong to the fourth subcircuit. The small resistors are chosen to be 0.1 ~ 0.001 ohm so that the conductances are 10 ~ 1000 mhos. When the entire circuit reaches the equilibrium point, $v_{n1} \approx v_7 \approx v'_9 \approx \tilde{v}_8$, $v_{n2} \approx v_8 \approx v'_8$, $v_{n3} \approx v_9 \approx \tilde{v}_9$, $v_{n4} \approx \bar{v}_8 \approx v'_7$, $v_{n5} \approx \bar{v}_9 \approx \tilde{v}_7$.
The matrices of the partitioned networks with the inserted resistors are formulated by the modified nodal approach and their structure is shown in Fig. 7.

When the Katzenelson-Gauss-Seidel method is applied to solve the matrix equation of this bipolar transistor circuit, the iterative procedure is as follow:

(a) n1 ~ n5 are taken as tearing nodes;

(b) initial guesses for $v_{n1} \sim v_{n5}$ are given;

(c) each subcircuit is separated such that each subcircuit appears to be driven by independent voltage sources $v_{n1} \sim v_{n5}$ as in Fig. 8;

(d) Katzenelson's algorithm is applied to solve each subcircuit at a set of guesses of $v_{n1} \sim v_{n5}$;

(e) use the solution to the subcircuit to update the set of $v_{n1} \sim v_{n5}$ by (28), and go to solve next subcircuit by Katzenelson's algorithm;

(f) after solving all of the subcircuits, obtain new set of $v_{n1} \sim v_{n5}$ in terms of (28);

(g) check the new set of $v_{n1} \sim v_{n5}$ with old set of guesses, if their differences are less than a given tolerance, stop iterating, otherwise, take the new set of $v_{n1} \sim v_{n5}$ as guesses and go to the step (d) and iterate again.

As was discussed above, if the new reduced diagonal blocks, which are generated from the diagonal blocks by eliminating internal node voltage variables in the corresponding subcircuits, are diagonally dominant or class M-matrices, then the Katzenelson-Gauss-Seidel method converges. The following are some examples which show how to generate the new blocks.

The nodal matrix of the subcircuit 1 in a given region is:

$$\begin{bmatrix} 4950 & 4924 & 0000 & 0000 & 0000 & 0000 & 0000 & -24.8 & -.011 \\ -7385 & 9873 & -24.87 & -.4e-6 & 0000 & 0000 & 0000 & -2462 & -1.12 \\ 0000 & -2480 & 4975 & -.4e-6 & 0000 & 0000 & -2462 & 0000 & 0000 \\ 0000 & 2462 & -2462 & 2488 & -2462 & -24.87 & 0000 & 0000 & 0000 \\ 0000 & 0000 & 0000 & -2462 & 4934 & -2462 & 0000 & 0000 & 0000 \\ 0000 & 0000 & 0000 & -24.87 & -2462 & 4949 & -2462 & 0000 & 0000 \\ 0000 & 0000 & -2462 & 0000 & 0000 & -2462 & 7486 & 0000 & 0000 \\ -24.8 & -2462 & 0000 & 0000 & 0000 & 0000 & 0000 & 1e+10 & 0000 \\ 2461 & -2462 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 1e+10 \end{bmatrix}$$

In the subcircuits (or blocks), $v_1 \sim v_6$ are not related to the other subcircuit (or,blocks), they are only internal node voltages which do not appear in the Gauss-Seidel iterative process. Using the method in Remark 3.1, we can obtain:

Let

$$A_{rr} = \begin{bmatrix} 4950 & 4924 & 0000 & 0000 & 0000 & 0000 \\ -7385 & 9873 & -24.87 & -.4e-6 & 0000 & 0000 \\ 0000 & -2480 & 4975 & -.4e-6 & 0000 & 0000 \\ 0000 & 2462 & -2462 & 2488 & -2462 & -24.87 \\ 0000 & 0000 & 0000 & -2462 & 4934 & -2462 \\ 0000 & 0000 & 0000 & -24.87 & -2462 & 4949 \end{bmatrix}$$

$$A_{rs} = \begin{bmatrix} 0000 & -24.8 & -.011 \\ 0000 & -2462 & -1.12 \\ -2462 & 0000 & 0000 \\ 0000 & 0000 & 0000 \\ 0000 & 0000 & 0000 \\ -2462 & 0000 & 0000 \end{bmatrix}$$

$$A_{sr} = \begin{bmatrix} 0000 & 0000 & -2462 & 0000 & 0000 & -2462 \\ -24.8 & -2462 & 0000 & 0000 & 0000 & 0000 \\ 2461 & -2462 & 0000 & 0000 & 0000 & 0000 \end{bmatrix}$$

$$A_{ss} = \begin{bmatrix} 7486 & 0000 & 0000 \\ 0000 & 1e+10 & 0000 \\ 0000 & 0000 & 1e+10 \end{bmatrix}$$

and the new block is $A'_{ii}=A_{ss}-A_{sr}A_{rr}^{-1}A_{rs}$, which is

$$\begin{bmatrix} .261e{+}4 & -.103e{+}2 & -.471e{-}2 \\ -.122e{+}2 & .100e{+}11 & -.113e{+}1 \\ -.667e{-}1 & -.119e{+}1 & .100e{+}11 \end{bmatrix}$$

This is a class M-matrix as well as diagonally dominant.

The nodal matrix of subcircuit 3 in a given region is

$$\begin{bmatrix} 4924 & 4924 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & -.17e{-}12 \\ -9847 & 9848 & -.46e{-}8 & -.17e{-}10 & 0000 & 0000 & 0000 & 0000 & -0.17e{-}10 \\ 0000 & -.46e{-}6 & 1.00 & -.17e{-}10 & 0000 & 0000 & -.17e{-}10 & 0000 & 0000 \\ 0000 & .46e{-}6 & -.46e{-}6 & 2487 & -2462 & -24.87 & 0000 & 0000 & 0000 \\ 0000 & 0000 & 0000 & -2462 & 4934 & -2462 & 0000 & 0000 & 0000 \\ 0000 & 0000 & 0000 & -24.87 & -2462 & 2486 & -.51e{-}3 & 0000 & 0000 \\ 0000 & 0000 & -.46e{-}6 & 0000 & 0000 & -.51e{-}3 & 100.0 & 0000 & 0000 \\ 2462 & -2462 & 0000 & 0000 & 0000 & 0000 & 0000 & 100.0 & 0000 \\ 2462 & -2462 & 0000 & 0000 & 0000 & 0000 & 0000 & 0000 & 100.0 \end{bmatrix}$$

which can be transformed into:

$$\begin{bmatrix} .100e{+}3 & .24e{-}21 & .15e{-}10 \\ -.666e{-}16 & .100e{+}3 & -.853e{-}1 \\ -.645e{-}15 & -.136e{-}10 & .100e{+}3 \end{bmatrix}$$

This new block is diagonally dominant.

We can also transform the other blocks into new blocks which are diagonally dominant and sometimes are also M-matrices, so that the Jacobian matrices of the entire circuit are M-matrix or diagonally dominant also. The spectral radii of the block Gauss-Seidel matrices are less than 1, and the Katzenelson-Gauss-Seidel method is guaranteed to be convergent.

Note that in the original circuit in Fig.5, there are no resistors connecting the subcircuits together, and $v_7=v'_9=\tilde{v}_8$, $v'_7=\bar{v}_8$, $\tilde{v}_7=\bar{v}_9$. Since small resistors are inserted between the subcircuits,

the accuracy of node voltages is affected. We take the absolute differences between v_7, v'_9 and $\tilde{v}_8$, and the *difference* of v'_7 and $\bar{v}_8$, and the difference of $\tilde{v}_7$ and $\bar{v}_9$ as the accuracy measurement, for instance, v_7=3.99, v'_9=3.89, the accuracy is 0.1. The following table discribes the relation between the conductance, the iterative number and the accuracy.

Table 2

conductance	iterative number	accuracy
10	13	0.1
100	33	0.01
1000	165	0.005

If fewer resistors are inserted as shown in Fig.6 (b), faster convergence rate is obtained, since the *subcircuit outputs become the* 'independent voltage sources', which are now applied to directly drive the next subcircuits.

Example 2

In this example which is an MOS circuit there is no need to insert small resistors in order to apply the Katzenelson-Gauss-Seidel method. Instead of attempting to obtain a bordered-block-diagonal matrix structure, as is dine in Example 1, the circuit is decomposed into dc-connected subcircuits and then analysis sequence is performed to order the equations into nearly power-block-diagonal form [12] The example circuit shown in Fig.9 is an MOS register circuit. In this circuit and in the following example circuit, the NMOS transistor will be used to illustrate the method. The transistor model is

not discussed here in detail; the mathematical equations for the channel current in the NMOS transistor can be simply expressed as [11]:

$$I_{ds}=K(V_{gs}-V_t)^2 - K(V_{gd}-V_t)^2 \tag{29}$$

where I_{ds} is the channel current from Source to Drain, V_{gs} is the voltage across Gate and Source, V_{gd} is the voltage across Gate and Drain, V_t is the threshold voltage. This yields the Ebers-Moll 'like' model of Fig.10(a). Both forward and reverse α must equal to unity in order for the gate current $I_g=0$, as it must be for an MOS transistor. Fig.10(b) shows the PWL characteristics which is the approximation of the quadratic characteristics for a driver MOS transistor. The PWL characteristics for a load MOS transistor is the same shape as that of the driver except that the origin point and the vertical axis are shifted. For digital applications it was found that only three segments are sufficient to describe the characteristics. The parameters for PWL characteristics are as follows:

Table 3 for driver transistor

segment	breakpoint	slope	intercept
1	1.0	1.0e-12	-1.0e-12
2	2.25	18.0e-6	-18.0e-6
3	∞	82.e-6	-154.e-6

Table 4 for load transistor

segment	breakpoint	slope	intercept
1	-2.0	1.0e-12	2.0e-12
2	-1.0	6.25e-6	12.5e-6
3	∞	18.75e-6	25.0e-6

The entire circuit is divided into 3 subcircuits, as in Fig.11, v_1 and v_2 are the input, $v_3 \sim v_6$ are to be solved. We first solve the inverter in Fig.11(a), and obtain v_6 which is the solution, since v_6 is affected by v_2 only. For the subcircuit in Fig.11(b), v_1, v_2 and v_6 are given, v_5 is not known and has to be guessed; then v_4 and v_3 are solved in terms of Katzenelson's algorithm. After v_4 is obtained, it becomes the input to the inverter in Fig.11(c) which is also solved by Katzenelson's algorithm. The new v_5 is compared with the guess; if they are the same, the solution is obtained; otherwise, go to the subcircuit (b), and start the next iteration, until convergence is obtained. In this example, the Jacobian matrices of the entire circuit are class M-matrix in most of the regions, but not always. For instance, when the initial guesses are:

$$
\begin{aligned}
v_1 &= 0.0, \\
v_2 &= 0.0, \\
v_3 &= 0.305, \\
v_4 &= 0.38e-8, \\
v_5 &= 5.0, \\
v_6 &= 0.305,
\end{aligned}
$$

the matrix is:

$$
\begin{bmatrix}
0.20e-11 & 0.00000 & 0.0000 & 0.0000 \\
0.0000 & 0.82e-4 & -0.10e-11 & 0.0000 \\
0.0000 & -0.10e-11 & 0.20e-11 & 0.0000 \\
0.0000 & 0.00000 & 0.00000 & 0.18e-4
\end{bmatrix}
$$

When they reach the solution:

$v_1 = 0.0,$
$v_2 = 0.0,$
$v_3 = 0.305,$
$v_4 = 0.305,$
$v_5 = 5.0,$
$v_6 = 5.0,$

the Jacobian matrix is:

$$\begin{bmatrix} 0.18e-4 & 0.00000 & 0.0000 & 0.0000 \\ 0.0000 & 0.16e-3 & -0.82e-4 & 0.0000 \\ 0.0000 & -0.82e-4 & 0.82e-4 & 0.0000 \\ 0.0000 & 0.00000 & 0.00000 & 0.18e-4 \end{bmatrix}$$

During the Gauss-Seidel iteration, the matrix elements change their values but they keep the same signs and diagoanal dominance is preserved, so that the Jacobian matrices are class M-matrices.

The Jacobian matrices are not class M-matrix in some PWL regions during Katzenelson's process. when other initial guesses are used; for instance, when the subcircuit (b) is been solving the node voltages are:

$v_1 = 0.0,$
$v_2 = 5.0,$
$v_3 = 3.0,$
$v_4 = 0.36e-7,$
$v_5 = 5.0,$
$v_6 = 0.305,$

the Jacobian matrix is:

$$\begin{bmatrix} 0.82e-4 & 0.00000 & 0.0000 & 0.0000 \\ 0.0000 & 0.18e-4 & -0.10e-11 & 0.64e-4 \\ 0.0000 & -0.10e-11 & 0.82e-4 & 0.0000 \\ 0.0000 & 0.00000 & 0.00000 & 0.18e-4 \end{bmatrix}$$

which is not class M-matrix

After the Katzenelson's process of subcircuit (b) reaches the solution: $v_3 = 0.305$, $v_4 = 0.37e-8$, with the other voltages the same, the Jacobian matrix becomes class M-matrix again:

$$\begin{bmatrix} 0.82e-4 & 0.00000 & 0.0000 & 0.0000 \\ 0.0000 & 0.82e-4 & -0.10e-11 & 0.0000 \\ 0.0000 & -0.10e-11 & 0.82e-4 & 0.0000 \\ 0.0000 & 0.00000 & 0.00000 & 0.18e-4 \end{bmatrix}$$

From the viewpoint of the Gauss-Seidel iterations, the Jacobian matrices are class M-matrix in the region where the solution exists and the Katzenelson-Gauss-Seidel method converges although the Jacobian matrix may not be class M-matrix in some regions.

<u>Example</u> 3

The third example is a large-scale MOS circuit-a programmable logic array (PLA), which is used to implement a traffic light controller [13]. The circuit, which is shown in Fig. 12, is partitioned into 42 subcircuits. Some feedback node voltages are assumed to be given by previous iterates, according to the Gauss-Seidel method. The subcircuits are solved by Katzenelson algorithm in a sequence which is obtained by topological properties. After three iterations, the solution is reached. The total computational time is 0.683 second. This example shows that the Katzenelson-Gauss-Seidel method is suitable for large-scale circuits.

Acknowledgment

The author wishes to thank Professor I. N. Hajj for his invaluable help and comments.

Appendix

Appendix 1

The following two proofs can be found in [6], which are repeated here for easy reference.

Proof of Theorem 2.2

Supposing starting points are X^o, $Y^o \in R^n$,

$$X^o=(x_1^o,\ldots,x_i^o,\ldots,x_1^o)^T,\quad Y^o=(y_1^o,\ldots,y_i^o,\ldots,y_1^o)^T$$

and mapping F can be expressed

$$\begin{aligned} f_1(X) &= b_1 \\ \vdots \quad &\quad \vdots \\ f_i(X) &= b_i \\ \vdots \quad &\quad \vdots \\ f_n(X) &= b_n \end{aligned}$$

As induction hypothesis, for some $K \geq 0$, $i \geq 1$,

$$X^o \leq X^k \leq V^* \leq Y^k \leq y^o \quad F(X)^k \leq B \leq F(Y)^k \tag{30}$$

$$x_j^k \leq x_j^{k+1} \leq y_j^{k+1} \leq y_j^k, \qquad j=1,\ldots,i-1. \tag{31}$$

For $k=0, i=1$, (30) holds, and (31) is empty, suppose (30) and (31) hold for k and $i-1$. By the off-diagonal antitone property, it follows that the functions

$$P(s)=f_i(x_1^{k+1},\ldots,x_{i-1}^{k+1},s,x_{i+1}^k,\ldots,x_n^k) \tag{32}$$

$$Q(s)=f_i(y_1^{k+1},\ldots,y_{i-1}^{k+1},s,y_{i+1}^k,\ldots,y_n^k) \tag{33}$$

satisfy

$$Q(s) \leq P(s)\ , \text{ and } s \in R^1 \tag{34}$$

and

$$Q(x_i^k) \leq P(x_i^k) \leq f_i(X^k) \leq b_i \leq f_i(Y^k) \leq Q(y_i^k) \leq P(y_i^k)$$

Let $P(\bar{x}_i^k) = b_i = Q(\bar{y}_i^k)$, by the strictly diagonally isotone property, we obtain

$$x_i^k \leq \bar{x}_i^k \leq \bar{y}_i^k \leq y_i^k.$$

where the relation $\bar{x}_i^k \leq \bar{y}_i^k$ is the consequence of (34)

For Gauss-Seidel method : $y_i^{k+1} = \bar{y}_i^k$

$$x_i^{k+1} = \bar{x}_i^k$$

then, $x_i^k \leq x_i^{k+1} \leq y_i^{k+1} \leq y_i^k$ holds for i=1,2,...,n and hence,

$$X^k \leq X^{k+1} \leq Y^{k+1} \leq Y^K.$$

Then, we obtain,

$$f_i(Y^{k+1}) \geq f_i(y_i^{k+1},\ldots,y_i^{k+1},y_{i+1}^k,\ldots)=b_i.$$

so,

$$F(Y^{k+1}) \geq B$$

Similarly,

$$F(X^{k+1}) \leq B$$

This completes the induction and hence the proof of (30) and (31).

Clearly, now the limits

$$\lim_{k\to\infty} X^k = X^* \leq Y^* = \lim_{k\to\infty} Y^k$$

exist and by the definition and the continuous mapping , it follows that $F(X)^* = F(Y)^* = B$, and $X^* = Y^*$

<u>Proof</u> of Theorem 2.3

For given $X^o, B \in R^n$, define

$$U^o = F^{-1}(\max[f_1(X^o),B_1],\ldots,\max[f_n(X^o),B_n])$$

$$V^o = F^{-1}(\min[f_1(X^o),B_1],\ldots,\min[f_n(X^o),B_n])$$

By the inverse isotonicity

$F(U^o) \geq B \geq F(V^o)$, means that $U^o \geq X^o \geq V^o$ and $U^o \geq X^* \geq V^o$

Let $\{U^k\}$, $\{V^k\}$ and $\{X^k\}$ denote the Gauss-Seidel iterative sequences starting from U^o, V^o and X^o respectively. By the property of strictly diagonally isotone and continuity of F, the solutions $\bar{u}_i$, $\bar{v}_i$ and $\bar{x}_i$ of equations

$$f_i(u_1^{k+1},\ldots,u_{i-1}^{k+1},\bar{u}^k,u_{i+1}^k,\ldots,u_n^k = b_i$$

$$f_i(v_1^{k+1},\ldots,v_{i-1}^{k+1},\bar{v}^k,v_{i+1}^k,\ldots,v_n^k = b_i$$

$$f_i(x_1^{k+1},\ldots,x_{i-1}^{k+1},\bar{x}^k,x_{i+1}^k,\ldots,x_n^k = b_i$$

$$i=1,\ldots,n,\ k=0,1,\ldots$$

exist and are unique and therefore the Gauss-Seidel sequences are well defined. Considering U^o, V^o as X^o and Y^o in theorem 2.2, we can obtain

$$V^o \leq V^{k+1} \leq V^k \leq \lim_{k\to\infty} V^k \leq X^* \leq \lim_{k\to\infty} U^k \leq U^{k+1} \leq U^k \leq U^o$$

$$F(V^k) \leq B \leq F(U^k)$$

For $\{X\}$, suppose that for some $k \geq 0$, $i \geq 1$

$$V^k \leq X^k \leq U^k,\ v_j^{k+1} \leq x_j^{k+1} \leq u_j^{k+1},\quad j=1,\ldots,i-1$$

then,

$$f_i(u_1^{k+1},\ldots,u_{i-1}^{k+1},\bar{u}_i,u_{i+1}^{k},\ldots,u_n^{k}) = b_i$$
$$= f_i(x_1^{k+1},\ldots,x_{i-1}^{k+1},\bar{x}_i,x_{i+1}^{k},\ldots,x_n^{k})$$
$$\geq f_i(u_1^{k+1},\ldots,u_{i-1}^{k+1},\bar{x}_i,u_{i+1}^{k},\ldots,u_n^{k})$$

together with the strict diagonal isotonicity of F, implies that

$$\bar{U}_i \geq \bar{X}_i$$

Similarly,

$$\bar{V}_i \leq \bar{X}_i$$

it follows that

$$v_i^{k+1} \leq x_i^{k+1} \leq u_i^{k+1}$$

holds for $i=1,\ldots,n$ and $k=0,1,\ldots$; this completes the induction, and by continuous mapping so the limit

$$\lim_{k\to\infty} X^k = X^*.$$

exists, the three sequences all converge to the unique solution.

Appendix 2

The PLA circuit is solved by means of the PREMOS program [12] which is modified to include the PWL analysis part. The following is the input data file for PREMOS. When the PWL analysis is needed, the word 'pwl' is added in the file and the program does PWL analysis for the DC equilibrium point.

```
PLA finite-state machine implementing the light controller
*subcircuit model card
model inv nor2 (5 1 10f 100f)
model nor3 andoi(5 5 1 10f 10f 10f 100f 0 3)
model nor4 andoi(5 5 1 10f 10f 10f 100f 0 4)
model notr1 trans(5 1 2 10f 100f 10f 50f 1 1)
model notr2 trans(5 1 2 10f 100f 10f 50f 2 1)
model notr4 trans(5 1 2 10f 100f 10f 50f 4 1)
model notr5 trans(5 1 2 10f 100f 10f 50f 5 1)
model clk1 source (4 1 10n 5n 10n 5n)
model clk2 source (5 0 5n 5n 5n 5n)
* AND plane
x1 11 17 19 1 nor3
x2 13 17 19 2 nor3
x3 12 14 17 19 3 nor4
x4 15 18 19 4 nor3
x5 16 18 19 5 nor3
x6 12 13 18 20 6 nor4
x7 11 18 20 7 nor3
x8 14 18 20 8 nor3
x9 15 17 20 9 nor3
x10 16 17 20 10 nor3
* OR plane
x11 5 6 7 8 9 21 56 28 notr5
x12 3 4 5 6 22 56 29 notr4
x13 3 5 7 8 10 23 56 30 notr5
x14 6 7 8 9 10 24 56 31 notr5
x15 4 5 25 56 32 notr2
x16 1 2 3 4 5 26 56 33 notr5
x17 9 10 27 56 34 notr2
* output registers
x18 28 35 55 49 notr1
x19 29 36 55 48 notr1
x20 30 30 37 inv
x21 31 31 38 inv
x22 32 32 39 inv
x23 33 33 40 inv
```

```
x24 34 34 41 inv
* input buffers
x25 57 42 55 45 notrl
x26 58 43 55 46 notrl
x27 59 44 55 47 notrl
* input registers
x28 45 45 50 inv
x29 46 46 51 inv
x30 47 47 52 inv
x31 48 48 53 inv
x32 49 49 54 inv
x33 50 50 11 inv
x34 45 45 12 inv
x35 51 51 13 inv
x36 46 46 14 inv
x37 52 52 15 inv
x38 47 47 16 inv
x40 53 53 17 inv
x41 48 48 18 inv
x42 54 54 19 inv
x43 49 49 20 inv
*input sources
va1 55 0 clk1 0 1 0 0 0 1 0 0 0 1 0 0 0 1
va2 56 0 clk1 0 0 0 1 0 0 0 1 0 0 0 1 0 0
va0 57 0 clk2 1 1 1 1 0 0 0 0 0 0 1 1 1 1 1 1
vb0 58 0 clk2 1 1 1 1 0 0 0 0 0 0 0 0 0 0 0 0
vc0 59 0 clk2 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1 1
*analysis requests
opt 1 1 3 1 1 1 1
contl 0 0 0
preset (35,5) (36,5) (55,1) (56,1)
time 1n 1n
dc
pwl
plot 55 56 42 43 44 35 36
plot 37 38 39 40 41 9 10
plot 1 2 3 4 5 6 7 8
*end 55 56 42 43 44 35 36
send 7 9 37 38 39 40 41
v+ 5
end
```

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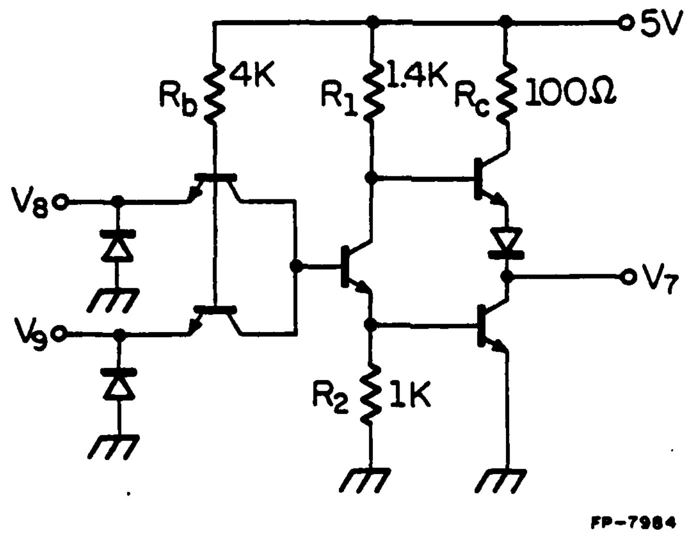

Fig. 1 TTL Nand Gate Circuit

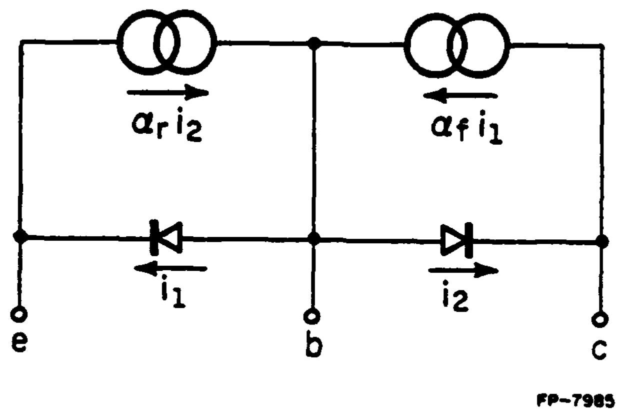

Fig. 2 Ebers-Moll Resistive Model of an npn Transistor

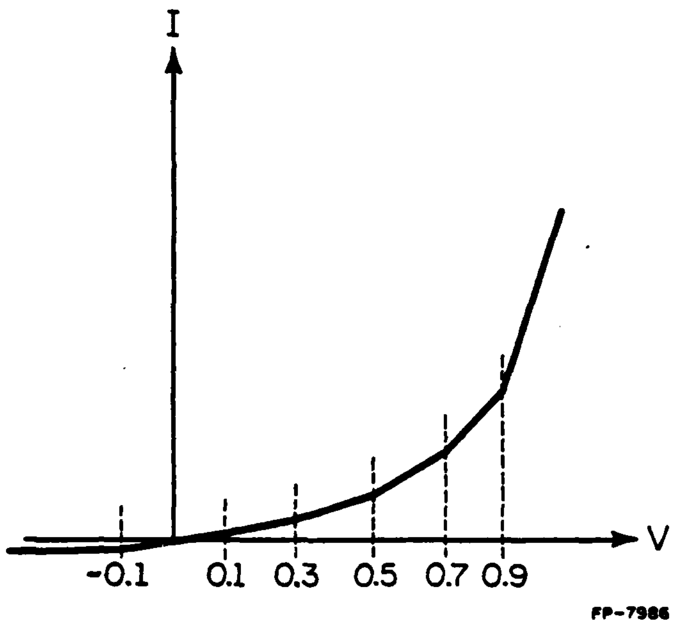

Fig. 3(a) PWL Characteristics of Bipolar Transistor

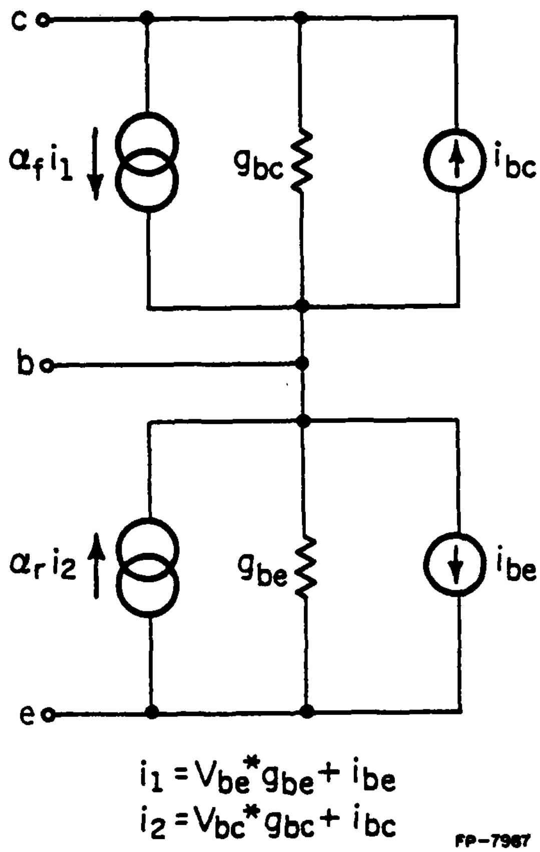

Fig. 3(b) PWL Equivalent Circuit for Bipolar Transistor

$$
\begin{array}{|ccccccccc|}
\hline
+ & - & & & & & & \pm & \pm \\
- & + & - & - & & & & & \\
 & \pm & + & - & & & - & & \\
 & \pm & - & + & - & - & & & \\
 & & & \pm & + & - & & & \\
 & & & \pm & - & + & - & & \\
 & & \pm & & & - & + & & \\
\pm & - & & & & & & + & \\
\pm & - & & & & & & & + \\
\hline
\end{array}
$$

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Fig. 4 The Sign Matrix of Nand Gate

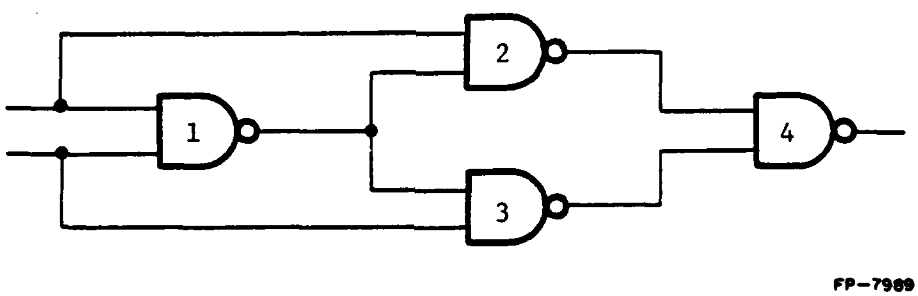

Fig. 5 The Circuit Consists of 4 Nand Gates

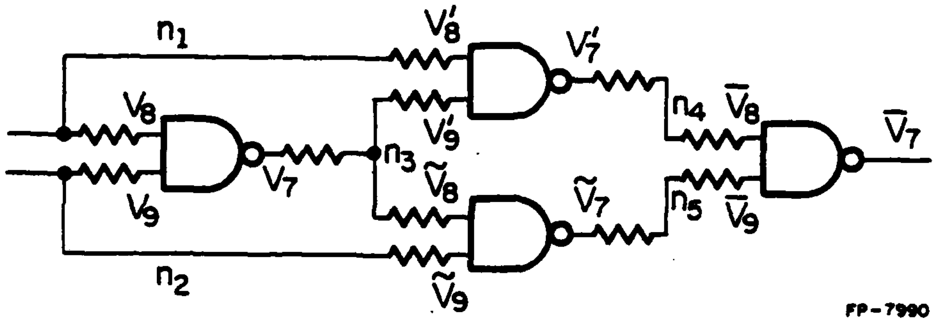

Fig. 6(a) The small Resistorts Inserted between the Gates

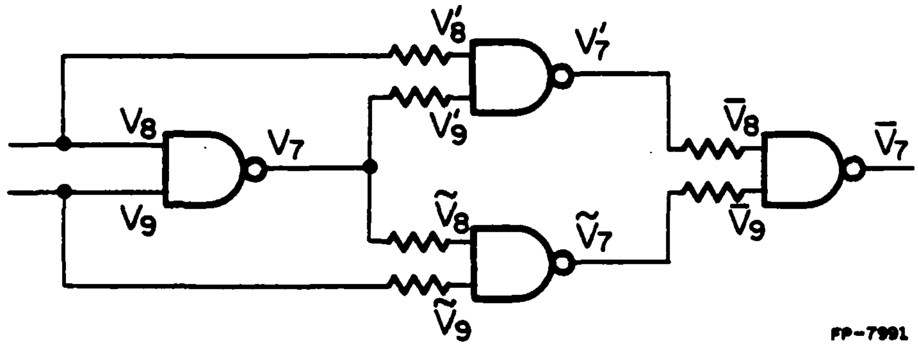

Fig. 6(b) Only One Resistor Inserted between the Gates

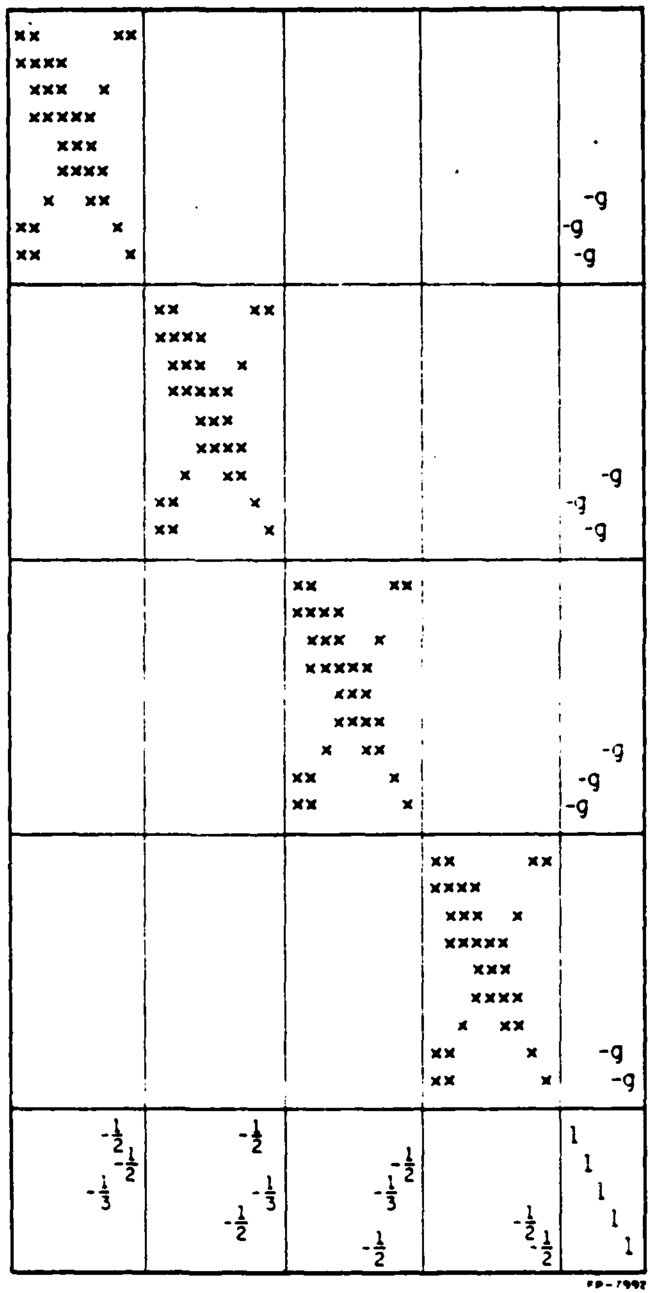

Fig. 7 The Block Structure of Matrix for Circuit in Fig. 6(a)

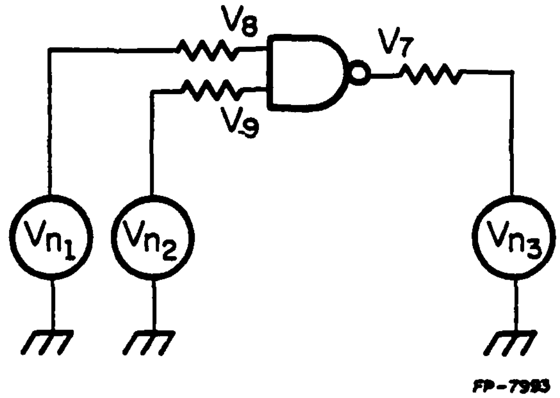

Fig. 8 Equivalent Sources connected with one Nand Gate

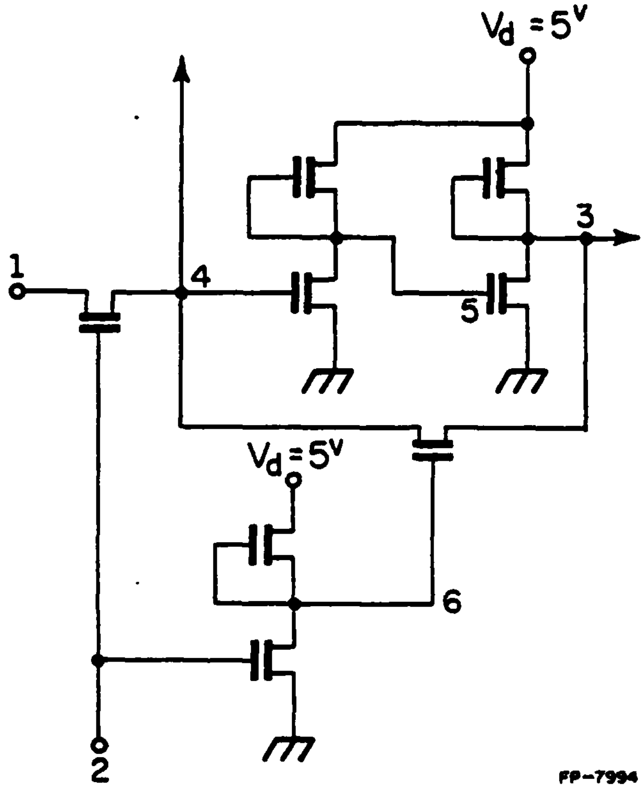

Fig. 9 MOS Register Circuit

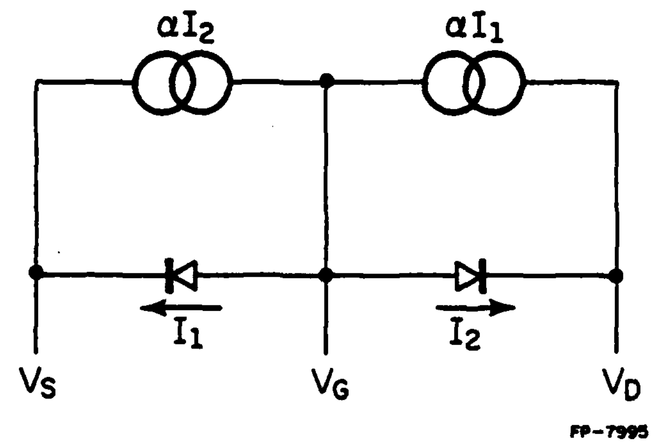

Fig. 10(a) Ebers-Moll 'like' MOS Model

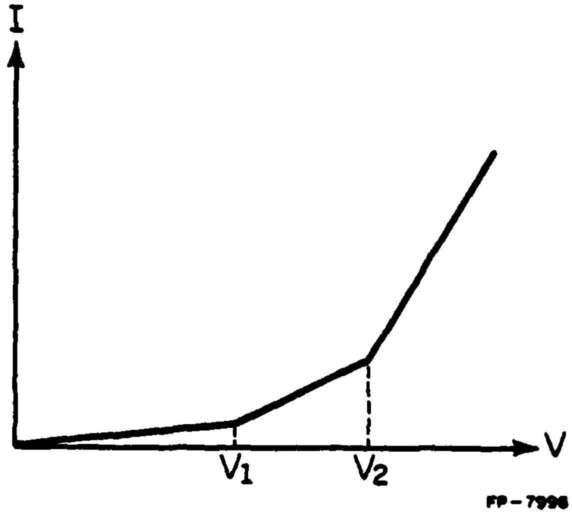

Fig. 10(b) PWL Characteristics of MOS Transistor

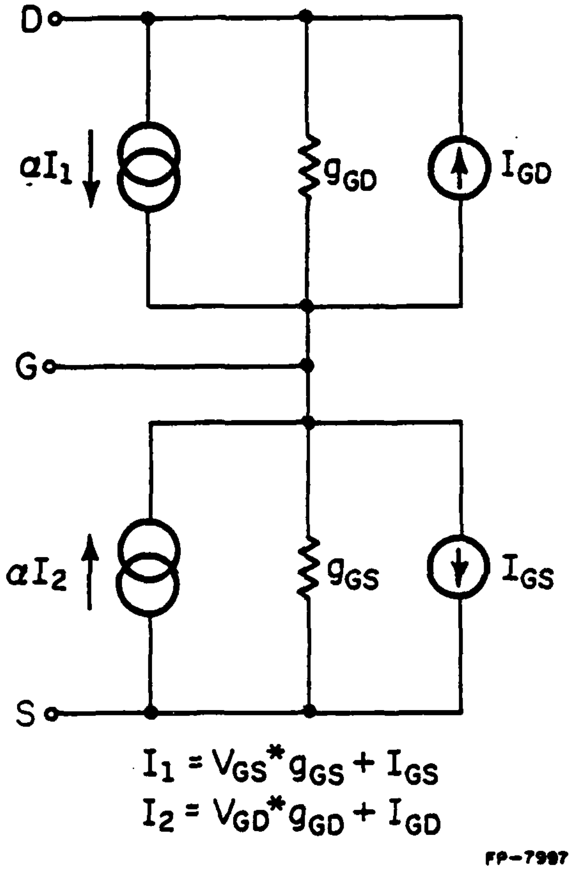

Fig. 10(c) PWL Equivlaent Circuit for MOS Transistor

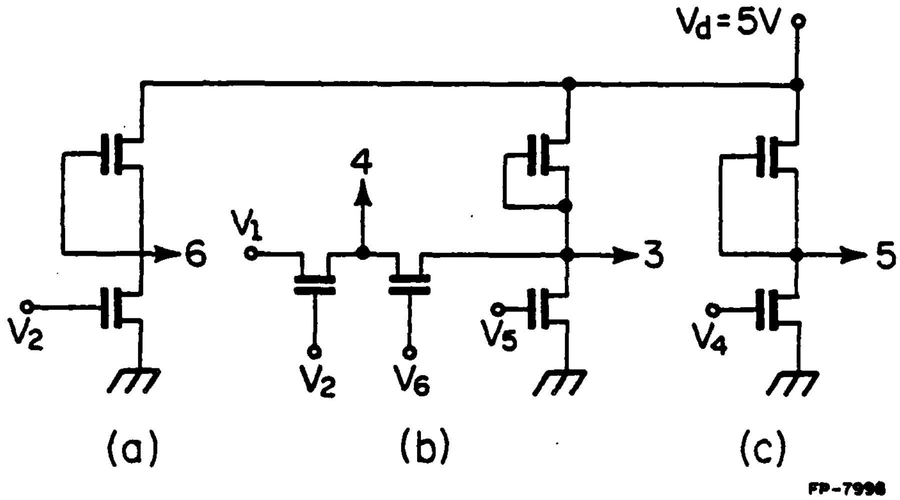

Fig. 11 The Subcircuits of MOS Register Circuit

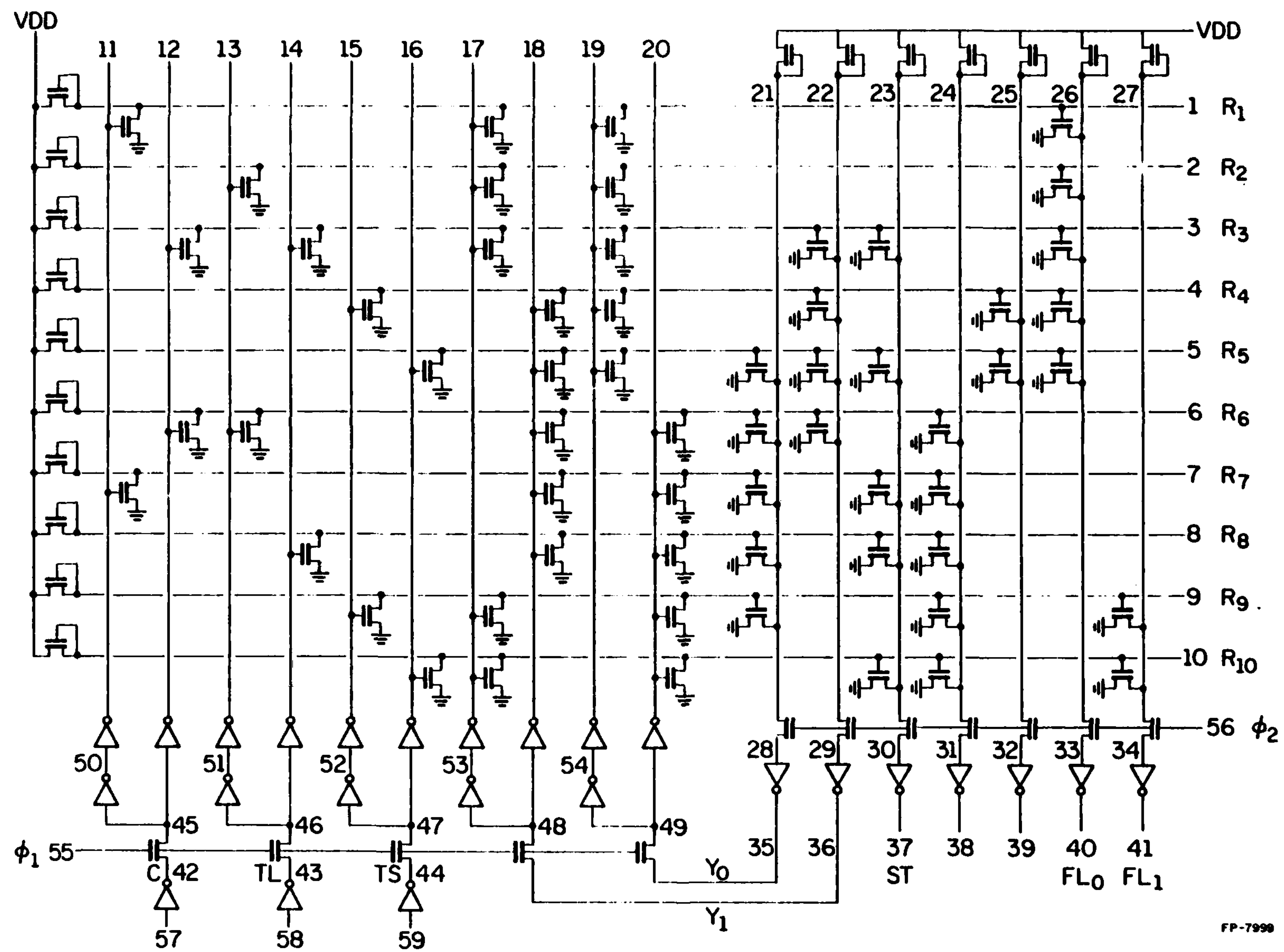

Fig. 12 Circuit Diagram of a PLA

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